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CS1303 April 2, 2021

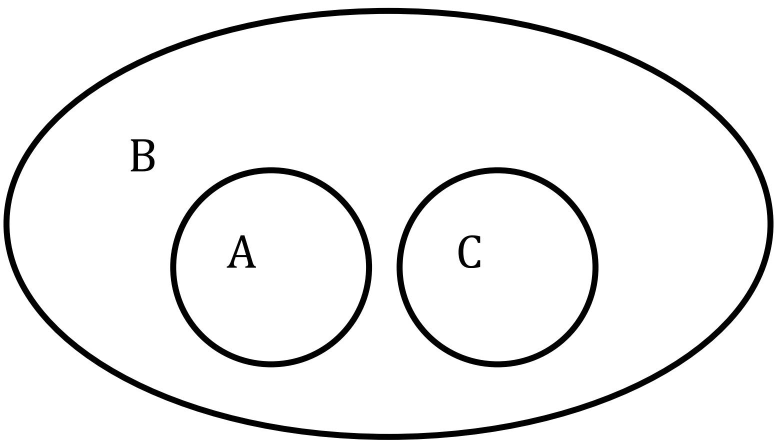
1.

a) {1, 3, 5, 6, 7, 9}
b) {3, 9}
c) {1, 2, 3, 4, 5, 6, 7, 8, 9}
d) ∅
e) {1, 5, 7}
f) {6}
g) {2, 3, 4, 6, 8, 9}
h) {6}

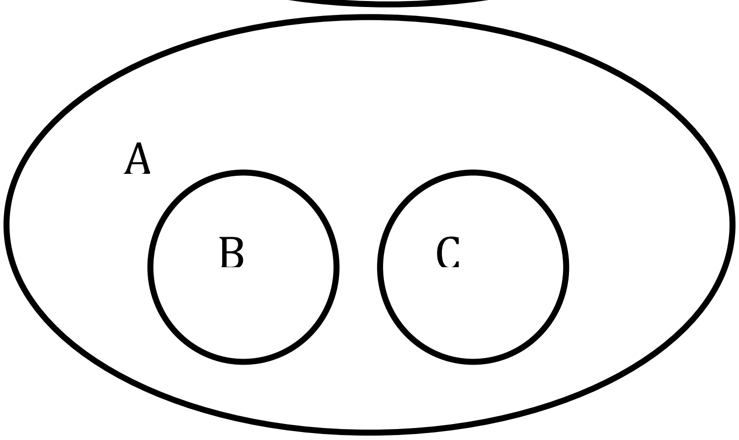
2.

a) {1110, 1111, 1000, 1001, 1100, 0100, 0111}
b) {1111}
c) {1110, 1000, 1001}
d) {1100, 0100, 0111}

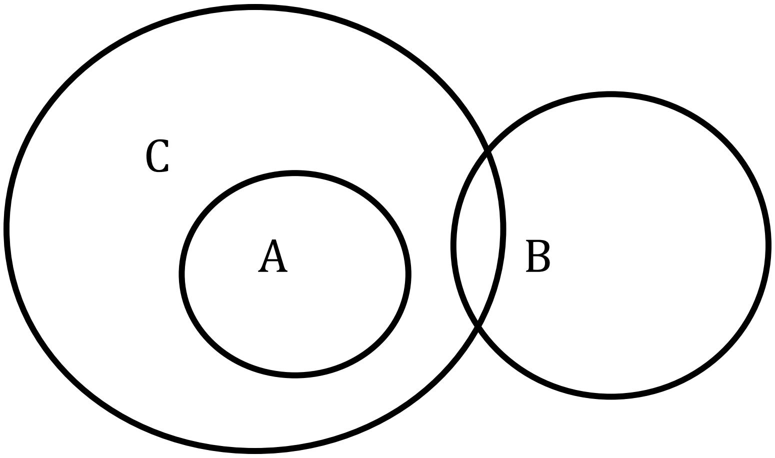
3.



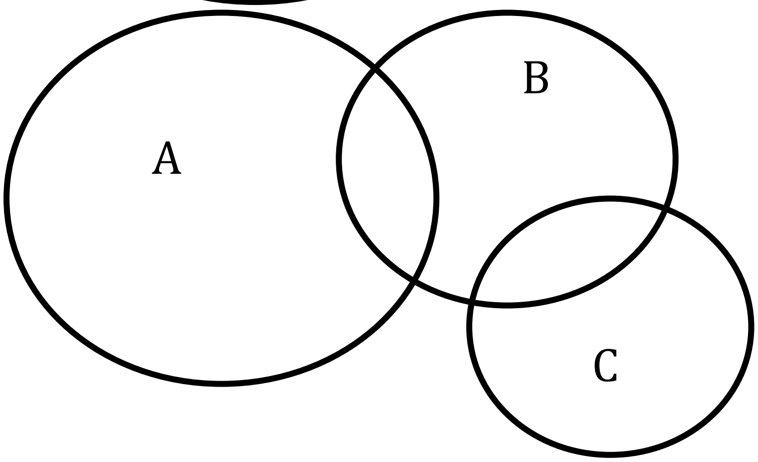
a)



b)



c)



d)

4.

a) {a, b} * {1, 2, 3, 4}
   {(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)}
b) {(a, 1), (a, 2), (b, 1), (b, 2)} ∪ {(a, 2), (a, 3), (b, 2), (b, 3)}
   {(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)}
c) {a, b} * {2}
   {(a, 2), (b, 2)}
d) {(a, 1), (a, 2), (b, 1), (b, 2)} ∩ {(a, 2), (a, 3), (b, 2), (b, 3)}
   {(a, 2), (b, 2)}

5. Yes, (A_0, A_1, A_2, A_3) is a partition of Z
As shown by the quotient remainder theorem, every integer can be represented as four forms $n = 4k \ or \ n = 4k + 1 \ or \ n = 4k + 2 \ or \ n = 4k + 3$ for some integer k. This means that any integer can only be in one of the sets, and that they must be in one of the sets A_0, A_1, A_2, A_3, which means that Z is a union of A_0, A_1, A_2, A_3

6.

a) Proof by contradiction

$P = \forall A, B, C \quad B \cap C \subseteq A \text{ -----> } (C - A) \cap (B - A) = \emptyset$

$\sim P$ is $\exists A, B, C\ (B \cap C \subseteq A \wedge \sim((C - A) \cap (B - A) = \emptyset)$

$\exists A, B, C\ (B \cap C \subseteq A \wedge (C - A) \cap (B - A) \neq \emptyset$

Suppose that $\sim p$ is true $((C - A) \cap (B - A) \neq \emptyset)$

$\exists X \in U$

$X \in (C - A) \wedge X \in (B - A) \neq \emptyset$

$(X \in C \wedge X \in A^C) \wedge (X \in B \wedge X \in A^C)$

$(X \in B) \wedge (X \in C) \wedge (X \in A^C)$

$(X \in B \cap C) \wedge (X \in A^C)$

Therefore $X \in B \cap C$ and $X \in A^C$

If $X \in B \cap C$ then $X \in A^C$

$(X \in B \cap C) \wedge (X \in A^C)$ = Contradiction

$\sim p$ is false

therefor p is true

b) Proof by contradiction

$P = \forall A, B, C, D\ A \cap C = \emptyset \text{ -----> } (A \times B) \cap (C \times D) = \emptyset$

$\sim P$ is $\exists A, B, C, D\ A \cap C = \emptyset \wedge ((A \times B) \cap (C \times D) \neq \emptyset)$

Suppose that $\sim P$ is true

$X \in (x, y)$

$(x, y) \in (A \times B) \cap (C \times D) \neq \emptyset$

$((x, y) \in A \times B) \wedge ((x, y) \in C \times D)$

$(x \in A \wedge y \in B) \wedge (x \in C \wedge y \in D)$

$x \in A \wedge y \in B \wedge x \in C \wedge y \in D$

$(x \in A \cap C) \wedge (y \in B \cap D)$

$(x \in A \cap C) \text{ -> } A \cap C = \emptyset$ and $A \cap C \neq \emptyset$ which is a contradiction

$\sim p$ is false

therefor p is true

7.

a) $A = \{1, 2, 3\}$
$B = \{2, 3 ,4\}$
$C = \{3, 4, 5\}$

$(A \cup B) \cap C = A \cup (B \cap C)$

$A \cup B = \{1, 2, 3, 4\}$
$\{1, 2, 3, 4\} \cap \{3, 4, 5\} = \{3, 4\}$
$\{3, 4\} = A \cup (B \cap C)$

$B \cap C = \{3, 4\}$
$\{1, 2, 3\} \cup \{3, 4\} = \{1, 2, 3, 4\}$
$\{3, 4\} = \{1, 2, 3, 4\}$ cannot be true.

b) $A = \{1, 2, 3\}$
$B = \{7, 8, 9\}$
$C = \{1, 2, 3\}$

$A \not\subset B$, $B \not\subset C$ but $A \subseteq C$